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# Common Gnoseological Meaning of Gödel and Caratheodory Theorems

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## Abstract

We will demonstrate that the *I. and the II. Caratheodory theorems* and their common formulation as the *II. Law of Thermodynamics* are physically analogous with the *real* sense of the Gödel's wording of his *I. and II. incompleteness theorems*. By using physical terms of the *adiabatic changes* the Caratheodory theorems express the properties of the *Peano Arithmetic inferential process* (and even properties of any *deductive and recursively axiomatic* inference generally); as such, they set the physical and then logical limits of any real inference (of the sound, not paradoxical thinking), which can run only on a physical/thermodynamic basis having been compared with, or translated into the formulations of the Gödel's proof, they represent the first historical and *clear* statement of gnoseological limitations of the deductive and recursively axiomatic inference and sound thinking generally. We show that **semantically understood** and with the language of logic and meta-arithmetics, the full meaning of the **Gödel proof expresses the universal validity of the II. law of thermodynamics and that the Peano arithmetics is not self-referential and is consistent**.<sup>1</sup>

**Keywords:** arithmetic formula, thermodynamic state, adiabatic change, inference

## 1. Introduction

To show that the real/physical sense of the Gödel incompleteness theorems—that the very real sense of them—is the meta-arithmetic-logical analog of the Caratheodory's claims about the *adiabatic system* (that they are the analog of the sense of the *II. Law of Thermodynamics*), we compare the states in the *state space* of an *adiabatic thermodynamic system* with *arithmetic formulas* and the *Peano inference* is compared with the *adiabatic changes* within this state space. The *whole set of the states* now *not achievable adiabatically* represents the existence of the states on an adiabatic path, but this fact is not expressible adiabatically. This property of which is the

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<sup>1</sup> The reader of the paper should be familiar with the Gödel proof's way and terminology; *SMALL CAPITALS* in the whole text mean the Gödel numbers and working with them. This chapter is based, mainly, on the [1–4]. This paper is the continuation of the lecture *Gödel Proof, Information Transfer and Thermodynamics* [4].

analog of the sense of *Gödel undecidable formula*. Nevertheless, any of these states, now not achievable adiabatically in the given state space (of the given adiabatic system), is achievable adiabatically *but* in the redefined and wider adiabatic system with its state space divided between adiabatic and not adiabatic parts again. These states (which are achievable only when the previous subsystem is part of the new actual system, both are consistent/adiabatic) represent arithmetic but not the Peano arithmetic formulas and also are bearing the property of their whole set. Also they can be axioms of the higher/superior inference including the previous one—the *general arithmetic inference* is further ruled by the same and repeated principle of widening the axiomatics and with same thermodynamic analogy using the redefined and widened new adiabatic system and its settings and with the same limitation by the impossibility to proof both the consistency of the given inferential system and, in our analogy, the adiabacity of its given adiabatic analog, by means of themselves. The consistency of the inferential system and adiabacity of its analog (and their abilities generally) are defined and proved by outer construction, outer limitations, and outer settings only (compare this our claim with the Gödel's claim for the Peano arithmetic inference “... in the Peano arithmetic system exists ...”).

#### Caratheodory common formulation of the II. P.T.:

*In the arbitrary vicinity of every state of the state space  $\Omega_{\mathcal{L}}$  of the adiabatic system  $\mathcal{L}$  exist states not reachable from the starting state adiabatically ( $[d]Q_{\text{Ext}}=0$ ) (or the states not reachable by the system at all).*

For the consistency of the Peano arithmetic theory  $\mathcal{T}_{\mathcal{PA}}$  the analog is expressed by:

#### Gödel incompleteness theorems:

*For the theory  $\mathcal{T}_{\mathcal{PA}}$  exists the true (“1”) CLAIM that either this CLAIM and its NEGATION is NOT PROVABLE within the system  $\mathcal{P}/\mathcal{T}_{\mathcal{PA}}$ .*

- CLAIM about the  $\mathcal{T}_{\mathcal{PA}}$  consistency especially -

*The CLAIM saying that theory  $\mathcal{T}_{\mathcal{PA}}$  is consistent is not PROVABLE by its means ( $\mathcal{P}$ ) - by itself.*

In our considerations, we use the states of the adiabatic system as the thermodynamic representation of the Peano arithmetically inferred formulas and the transition between the stats is then the thermodynamic model of the Peano arithmetic inference step, the consistency of the Peano arithmetics is represented by the adiabacity of the modeling thermodynamic system.

## Peano Axioms/Inference Rules in the System $\mathcal{P}$ /Theory $\mathcal{T}_{\mathcal{PA}}$ .

1/ $\mathcal{P}$	$\mathbb{N}_0 = \mathbb{N} \cup \{0\};$
2	$\forall_{x \in \mathbb{N}_0} [\exists_{y \in \mathbb{N}} [y = f(x)]];$
3/ $\mathcal{P}$	$\forall_{x \in \mathbb{N}_0} [0 \neq f(x)];$
4	$\forall_{x \in \mathbb{N}_0} [[f(x) \neq f(y)] \Rightarrow (x \neq y)];$
5/ $\mathcal{P}$	axiom/axiomatic schema of the <b>mathematical induction</b> : $[[\varphi(0) \wedge \forall_{x \in \mathbb{N}_0} [\varphi(x) \Rightarrow \varphi[f(x)]]] \Rightarrow \forall_{x \in \mathbb{N}_0} [\varphi(x)]]$
<b>Inference rule <i>Modus Ponens</i></b>	
$\frac{\vdash b, \vdash (b \Rightarrow c)}{\vdash c}, \quad c - \text{immediate consequence of } b$	
<b>Inference rule <i>Generalization</i></b>	
$\frac{a}{\forall_v a}, \quad \frac{a}{x \Pi(a)}, \quad \text{better} \quad \frac{\vdash a}{\vdash \forall_v a}, \quad \frac{\vdash a}{\vdash [x \Pi(a)]}, \quad \forall_v a/x \Pi(a) - \text{immediate consequence of } a$	

- ♣ “1” - arithmeticity of the  $\mathcal{P} \cong$  adiabaticity of the  $\mathcal{L}/\mathcal{D}_{\mathcal{L}}$ .
- ♣ Consistent  $\mathcal{T}_{\mathcal{PA}}$  inference within  $\mathcal{P} \cong$  moving along trajectories  $1_{\mathcal{D}_{\mathcal{L}}}$  in  $\mathcal{D}_{\mathcal{L}}/\mathcal{L}$ .
- ♣ The states on the adiabatic trajectories, also irreversible, then model the consistently inferred/inferable *PA-FORMULAS*.

**Remark:** Any inference within the system  $\mathcal{P}^2$  sets the  $\mathcal{T}_{\mathcal{PA}}$ -theoretical relation<sup>3</sup> among its formulae  $a_{[\cdot]}$ . This relation is given by their gradually generated *special sequence*  $\vec{a} [a_1, \dots, a_q, \dots, a_p, \dots, a_k, a_{k+1}]$ , which is the *proof* of the latest inferred formula  $a_{k+1}$ . By this, the *unique* arithmetic relation between their *Gödel numbers*, *FORMULAE*  $x_{[\cdot]}, x_{[\cdot]} = \Phi(a_{[\cdot]})$ , is set up, too. The gradually arising *SEQUENCE* of *FORMULAE*  $x = \Phi(\vec{a})$  is the *PROOF* of its latest *FORMULA*  $x_{k+1}$ .

Let us assume that the given sequence  $\vec{a} = [a_{01}, a_{02}, \dots, a_o, \dots, a_q, \dots, a_p, \dots, a_k, a_{k+1}]$  is a special one, and that, except of axioms (axiomatic schemes)  $a_{01}, \dots, a_o$ , it has been generated by the correct application of the rule *Modus Ponens* only.<sup>4</sup>

Within the process of the (*Gödelian*) *arithmetic-syntactic analysis* of the latest formula  $a_{k+1}$  of the proof  $\vec{a}$ , we use, from the  $\vec{a}$ -selected, (special) subsequence  $\vec{a}_{q,p,k+1}$  of the formulae  $a_q, a_p, a_{k+1}$ . The formulae  $a_q, a_p$  have already been derived, or they are axioms. It is valid that  $q, p < k+1$ , and we assume that  $q < p$ ,

$$\begin{aligned} \vec{a}_{q,p,k+1} &= [a_q, a_p, a_{k+1}], \quad a_p \cong a_q \supset a_{k+1}, \quad \vec{a}_{q,p,k+1} = [a_q, a_q \supset a_{k+1}, a_{k+1}], \\ x &= \Phi(\vec{a}) = \Phi([\Phi(a_1), \Phi(a_2), \dots, \Phi(a_q), \dots, \Phi(a_p), \dots, \Phi(a_k), \Phi(a_{k+1})]) \\ &= \Phi(\vec{x}) = \Phi(x_1) * \Phi(x_2) * \dots * \Phi(x_q) * \dots * \Phi(x_q) * \dots * \Phi(x_k) * \Phi(x_{k+1}) \\ l(x) &= l[\Phi(\vec{x})] = l[\Phi(\vec{a})] = k+1, \\ x_{k+1} &= \Phi(a_{k+1}) = l[\Phi(\vec{a})] Gl \Phi(\vec{a}) = (k+1) Gl x \\ x_p &= \Phi(a_p) = \Phi(a_p \supset a_{k+1}) = q Gl \Phi(\vec{a}) * \Phi(\supset) * l[\Phi(\vec{a})] Gl \Phi(\vec{a}) \\ &= q Gl x Imp [l(x)] Gl x \\ x_q &= \Phi(a_q) = q Gl \Phi(\vec{a}) = q Gl x \end{aligned}$$

<sup>2</sup> Formal arithmetic inferential system.

<sup>3</sup> Peano Arithmetics Theory.

<sup>4</sup> For simplicity. The ‘real’ inference is applied to the formula  $a_{i+1}$  for  $i = o$ .

Checking the *syntactic and  $\mathcal{T}_{\mathcal{PA}}$ -theoretical correctness* of the analyzed chains  $a_i$ , as the formulae of the system  $\mathcal{P}$  having been generated by inferring (*Modus Ponens*) within the system  $\mathcal{P}$  (in the theory  $\mathcal{T}_{\mathcal{PA}}$ ), and also the special sequence of the formulae  $\vec{a}$  of the system (theory  $\mathcal{T}_{\mathcal{PA}}$ ), is realized by checking the *arithmetic-syntactic* correctness of the notation of their corresponding *FORMULAE* and *SEQUENCE of FORMULAE*, by means of the relations *Form*( $\cdot$ ), *FR*( $\cdot$ ), *Op*( $\cdot, \cdot, \cdot$ ), *Fl*( $\cdot, \cdot, \cdot$ ) “called” from (the sequence of procedures) relations *Bew*( $\cdot$ ),  $(\cdot)B(\cdot)$ , *Bw*( $\cdot$ );<sup>5</sup> the core of the whole (Gödelian) arithmetic-syntactic analysis is the (procedure) relation of *Divisibility*,

$$\text{Form}[\Phi(a_i)] = \text{"1"/"0"}, \quad \text{FR}\left[\Phi\left(\overrightarrow{a_1^{i+1}}\right)\right] = \text{"1"/"0"}, \quad 0 \leq i \leq k$$

$$\text{Op}[x_k, \text{Neg}(x_q), x_{k+1}] = \text{Op}[\Phi(a_p), \Phi[\sim(a_q)], \Phi(a_{k+1})] = \text{"1"/"0"}$$

$$\text{Fl}[(k+1)\text{Gl}x, p\text{Gl}x, q\text{Gl}x] = \text{"1"/"0"}$$

$$xBx_{k+1} = \text{"1"/"0"}, \quad \text{Bew}(x_{k+1}) = \text{"1"/"0"};$$

$$\Phi(a_p) \parallel 23^{3\text{Gl}\Phi(\overrightarrow{a_{q,p,k+1}})} \quad \& \quad \Phi(a_p) \parallel 7^{1\text{Gl}\Phi(\overrightarrow{a_{q,p,k+1}})} = \text{"1"/"0"}$$

## 2. Gödel theorems

**Remark:** The expression  $Sb\left(\begin{matrix} u_1 & u_2 \\ t & \\ Z(x) & Z(y) \end{matrix}\right)$  or the expression

$Sb\left(\begin{matrix} 17 & 19 \\ t & \\ Z(x) & Z(y) \end{matrix}\right)$  represents the result value of the Gödel number  $t[Z(x), Z(y)]$ ,

which is coding the (constant) claim  $T(x, y) \text{ z } PM$  has been generated by the substitution of  $x$  a  $y$  instead of the free variables  $X$  and  $Y$  in the function  $T(X, Y)$  from  $PM$  with its Gödelian code  $t(u_1, u_2)$  in the (arithmetized)  $\mathcal{P}$ ,

$$Sb\left(\begin{matrix} u_1 & u_2 \\ t & \\ Z(x) & Z(y) \end{matrix}\right) = Sb\left(\begin{matrix} 17 & 19 \\ t & \\ Z(x) & Z(y) \end{matrix}\right)$$

♣ Into the *VARIABLES*, we substitute the *SIGNS* of the same *type* but the introduction of the term *admissible substitution* itself is not supposing it wordly.

- Then it is possible to work even with the expressions not grammatically correct and thus with such chains, which are not *FORMULAE* of the system  $\mathcal{P}$  (and thus not belonging into the theory  $\mathcal{T}_{\mathcal{PA}}$ ).

Then the substitution function  $Sb\left(\begin{matrix} \dots \\ \cdot \\ \dots \end{matrix}\right)$  is not possible, within the frame

of the inference in the system  $\mathcal{P}$ , be used isolately as an arbitrarily performed number manipulation—in spite of the fact that it is such number manipulation really. It is used only and just within the frame of the language  $\mathcal{L}_{\mathcal{P}}$  and, above all, within the frame of the conditions specified by the *právé a jenom*

<sup>5</sup> *Formula*, *Reihe von Formeln*, *Operation*, *Folge*, *Glied*, *Beweis*, *Beweis*, see Definition 1–46 in [5–7] and by means of all other, by them ‘called’, relations and functions (by their procedures).



**INFERENCE of the elements of the language**  $\mathcal{L}_{\mathcal{T}_{PA}}$  only (and thus in the more limited way).

Others than/semantically (or by the type) homogenous application of the substitution function is not within the right inference/INFERENCE within the system  $\mathcal{P}$  possible.<sup>6</sup>

## 2.1 The Gödel UNDECIDABLE CLAIM's construction

◆ Let the Gödel numbers  $x$  and  $y$  be given. The number  $x$  is the *SEQUENCE OF FORMULAE* valid and  $y$  is a *FORMULA* of  $\mathcal{P}$ . We define the valid constant relation  $Q(x, y)$  from the  $Q(X, Y)$  for given values  $x$  and  $y$ ,  $X:=x$ ,  $Y:=y$ ;  $17 = \Phi(X)$ ,  $19 = \Phi(Y)$ ,<sup>7,8</sup>

$$Q(x, y) \equiv \overline{x B_{\kappa} \left[ Sb \left( \begin{array}{c} 19 \\ y \\ Z(p) \end{array} \right) \right]} \equiv Bew_{\kappa} \left[ Sb \left( \begin{array}{cc} 17 & 19 \\ q & \\ & Z(x) \quad Z(y) \end{array} \right) \right]$$

$$q[Z(x), Z(y)] = \Phi[Q(x, y)], \quad \overline{x B_{\kappa} y'} \equiv Bew_{\kappa}(y') = Bew_{\kappa}[y[Z(y)]] = Bew_{\kappa}[q[Z(x), Z(y)]]$$

(1)

◆ Now we put  $p = 17Gen \ q, q = q(17, 19) \ [q(17, 19) \triangleq Q(X, Y)]$  and then,

$$p = 17Gen \ q(17, 19) = \Phi[\forall_{x \in \mathbb{X}} | Q(x, Y)] \triangleq Q(\mathbb{X}, Y) \triangleq Q(\mathbb{N}_0, Y) \quad (2)$$

The meta-language symbol  $Q(\mathbb{X}, Y)$  or  $Q(\mathbb{N}_0, Y)$  is to be read: **No**  $x \in \mathbb{X}(\mathbb{N}_0)$  **is in the**  $\kappa$ -**INFERENCE relation to the variable**  $Y$  (to its space of values  $\mathbb{Y}$ ).

◆ Further, with the Gödel substitution function, we put  $q[17, Z(p)] = r(17) = r$ ,

$$r := Sb \left( \begin{array}{c} 19 \\ q \\ Z(p) \end{array} \right) \text{ and then } r = Sb \left( \begin{array}{cc} 19 & \\ q(17, 19) & \\ & Z(p) \end{array} \right) = r(17) = \Phi[Q(X, p)] \quad (3)$$

The Gödel number  $r$  is, by the substitution of the *NUMERAL*  $Z(p)$ , **supposedly only** (by [5–7]) the *CLASS SIGN* with the *FREE VARIABLE*  $17(X)$ ; with the values  $p$ , the  $r$  contains the feature of *autoreference*,

$$r = r(17) = q[17, Z(p(19))] = q[17, Z[17Gen \ q(17, 19)]] \triangleq Q(X, p)$$

$$= \Phi[Q[X, \Phi[\forall_{x \in \mathbb{X}} | Q(x, Y)]]]_{Y:=p} \triangleq Q[X, \Phi[Q(\mathbb{X}, Y)]] \triangleq Q[X, \Phi[Q(\mathbb{N}_0, Y)]]$$

(4)

◆ Within the Gödel number/code  $q, q = q[17, 19]$ , we perform the substitution  $Y := p$  and then  $X := x$  and write

<sup>6</sup> Substitution function  $Sb \left( \begin{array}{c} \cdots \\ \cdot \\ \cdots \end{array} \right)$  is, in this way, similar to the computer *machine instruction* which itself, is always able to realize its operation with its operands on the arbitrary storage place, but practically it is always applicated within the limited *address space* and within the given *operation regime/mode* of the computer's activity only (e.g. regime/mode *Supervisor* or *User*).

<sup>7</sup>  $\Phi$  and  $Z$  represents the *Gödel numbering* and  $Sb$  the *Substitution*,  $B, Bew$  the *PA-arithmetic Proof*.

<sup>8</sup> Following the Gödel Proposition V (the first part) [5–7].

$$\begin{aligned}
 r[Z(x)] &= Sb \left( \begin{array}{cc} 17 & 19 \\ q(17, 19) & \\ & Z(x) \quad Z(p) \end{array} \right) = Sb \left( \begin{array}{cc} 17 & \\ q[17, Z(p)] & \\ & Z(x) \end{array} \right) \\
 &= \Phi[Q[x, \Phi[Q(\mathbb{X}, Y)]]] = \Phi[Q[x, \Phi[Q(\mathbb{N}_0, Y)]]] = \Phi[Q(x, p)]
 \end{aligned} \tag{5}$$

With the great quantification of  $r[Z(x)]$  by  $Z(x)$  by the *VARIABLE*  $X$  (17), we have (similarly as in [4, 8]),

$$\begin{aligned}
 Z(x)Gen r[Z(x)] &= 17Gen q[17, Z[17Gen q(17, 19)]] = 17Gen r(17) = 17Gen r \\
 &\triangleq \Phi[\forall_{x \in X} |\Phi[Q[x, \Phi[\forall_{x \in X} |Q(x, Y)]]]] = \Delta Q[X, \Phi[Q(X, Y)]] = Q[\mathbb{N}_0, \Phi[Q(\mathbb{N}_0, Y)]]
 \end{aligned} \tag{6}$$

## 2.2 Gödel theorems

**I. Gödel theorem** (corrected semantically by [3, 9, 10]) claims that  
 ♣ **for every recursive and consistent CLASS OF FORMULAE  $\kappa$  and outside this set there is such true (“1”) CLAIM  $r$  with free VARIABLE  $v$  [ $r \triangleq r(v)$ ] that neither PROPOSITION  $vGen r$  nor PROPOSITION  $Neg(vGen r)$  belongs to the set  $Flg(\kappa)$ ,**

$$[vGen r \notin Flg(\kappa)] \ \& \ [Neg(vGen r) \notin Flg(\kappa)] \tag{7}$$

**FORMULA  $vGen r$  and  $Neg(vGen r)$  are not  $\kappa$ -PROVABLE—FORMULA  $vGen r$  is not  $\kappa$ -DECIDABLE.** They both are elements of inconsistent (meta)system  $\mathcal{P}^*$ .

**II. Gödel theorem** (corrected semantically according to [3, 9, 10]) claims that  
 ♣ **if  $\kappa$  is an arbitrary recursive and consistent CLASS OF FORMULAE, then any CLAIM saying that CLASS  $\kappa$  is consistent must be constructed outside this set, and for this fact it is not  $\kappa$ -PROVABLE.**

- Outside<sup>9</sup> the consistent system  $\mathcal{P}_\kappa$ , there is a true (“1”) formula,<sup>10</sup> the **ARITHMETIZATION** of which is  $\kappa$ -UNPROVABLE FORMULA  $17Gen r$ .<sup>11</sup>

◆ The fact that the recursive CLASS OF FORMULAE  $\kappa$  (now *PA—Peano Arithmetic* especially) is consistent, is tested by unary **relation**  $Wid(\kappa)$ , (die *Widerspruchsfreiheit, Consistency*) [5–7],

$$Wid(\kappa) \sim (Ex) \left[ Form(x) \ \& \ \overline{Bew_\kappa(x)} \right] \tag{8}$$

- a class of FORMULAE  $\kappa$  is consistent  $\stackrel{\text{Def}}{\Leftrightarrow}$  there exists at least one FORMULA  $x$  [PROPOSITION  $x$  ( $x = 17Gen r$ )], which is  $\kappa$ -UNPROVABLE.

## 3. Caratheodory theorems

**I. Caratheodory’s theorem** ( $\Rightarrow$ ) says that: ◇ If the **Pfaff form** has an integration factor, **then there are**, in the arbitrary vicinity of any arbitrarily chosen and fixed point

<sup>9</sup> Far from (!) “In ...” in [5–7]

<sup>10</sup> Far from “... [PA-]arithmetic and sentencial/SENTENCIAL” in [5–7].

<sup>11</sup> Any attempt to prove/TO PROVE it (to infer/to TO INFER it) in the system  $\mathcal{P}_\kappa$  assumes or leads to the requirement for inconsistency of the consistent (!) system  $\mathcal{P}_\kappa$  (in fact we are entering into the inconsistent metasystem  $\mathcal{P}^*$  - see the real sense [4, 9] of the Proposition V in [5–7]).

$P$  of the hyperplane  $\mathcal{R} [P \in \mathcal{R} [(x_i)_{i=1}^n] = \text{const.}]$ , **such points which**, from this point  $P$ , **are inaccessible** along the path satisfying the equation  $dQ = 0$ .

II. Caratheodory theorem ( $\Leftarrow$ ) says that:  $\Diamond$  If the Pfaff form  $\delta Q = \sum_{i=1}^n X_i dx_i$ , where  $X_i$  are continuously differentiable functions of  $n$  variables (over a simply continuous area), has such a property that in the arbitrary vicinity of any arbitrarily chosen and fixed point  $P$  of the hyperplane  $\mathcal{R} [P \in \mathcal{R} [(x_i)_{i=1}^n] = \text{const.}]$ , **there exists such points which**, from  $P$ , **cannot be accessible along the path satisfying the equation  $dQ = 0$** , then this form is **holonomous**; **it has or it is possible to find an integration factor for it**.

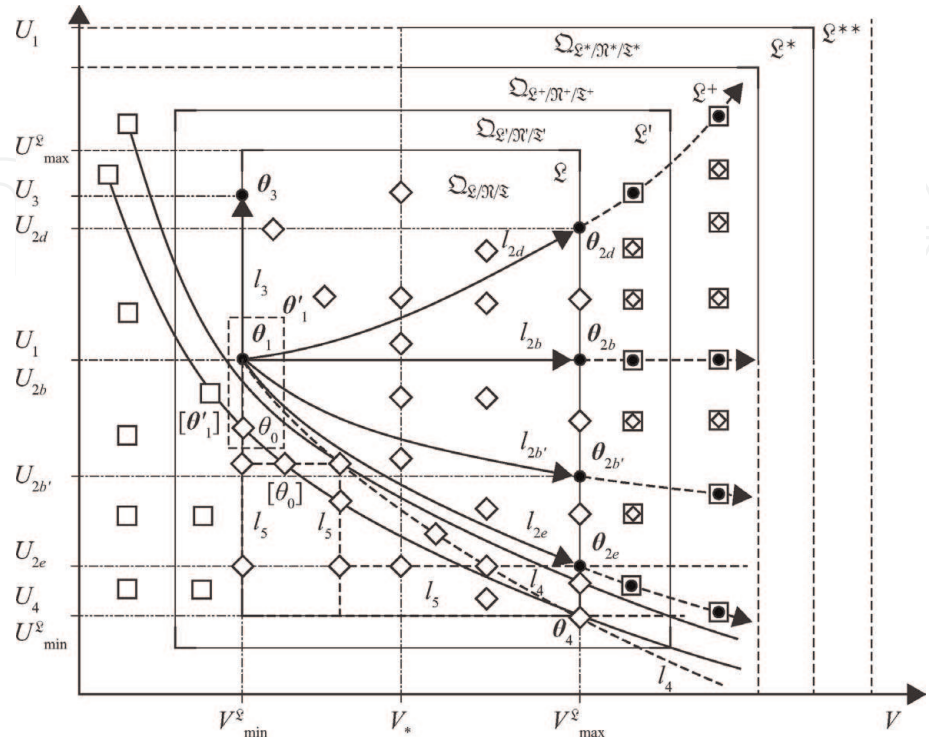
Caratheodory formulation of the II. Law of Thermodynamics ( $\Leftrightarrow$ ) claims that:

$\Diamond$  In the arbitrary vicinity of every state of the state space of the adiabatic system, **there are such states that, from the given starting point, cannot be reached along an adiabatic path** (reversibly and irreversibly), or such states which the system cannot reach at all, see the **Figure 1**.

**Remark:** Now the symbol  $Q$  denotes that heat given to the state space of the thermodynamic system from its outside and directly;  $Q \triangleq Q_{Ext}$ ; along paths  $l_{2b}$ ,  $l_{2b'}$ ,  $l_{2d}$ ,  $l_{2e}$ ,  $l_3$  is  $Q_{Ext} = 0$ ,  $\Delta Q_{Ext} = 0$ ,  $dQ_{Ext} = 0$ .

**♣ The states'  $\theta_{[\cdot]}^{\mathcal{L}}$  changes in the adiabatic system  $\mathcal{L}/\mathcal{D}_{\mathcal{L}}$ , along the trajectories  $l_{\mathcal{D}_{\mathcal{L}}}$  are expressible regularly:**

$l_{2b}$ isothermic irreversible,	$\theta_1^{\mathcal{L}} \rightarrow \Delta A_{1,2e} \theta_{2e}^{\mathcal{L}}$
$l_{2b'}$ adiabatic irreversible,	$\theta_1^{\mathcal{L}} \rightarrow \Delta A_{1,3} \theta_3^{\mathcal{L}}$
$l_{2d}$ izobaric irreversible,	$\theta_1^{\mathcal{L}} \rightarrow \Delta A_{1,2b} \theta_{2b}^{\mathcal{L}}$
$l_{2e}$ izentropic reversible,	$\theta_1^{\mathcal{L}} \rightarrow \Delta A_{1,2b'} \theta_{2b'}^{\mathcal{L}}$
$l_3$ izochoric irreversible,	$\theta_1^{\mathcal{L}} \rightarrow \Delta A_{1,2d} \theta_{2d}^{\mathcal{L}}$
	$\theta_1^{\mathcal{L}} \rightarrow \lambda \theta_1^{\mathcal{L}}$



**Figure 1.**  
Adiabatic changes of the state of the system  $\mathcal{L}$ , illustration.



Through the state space of *FORMULAE* of the system  $\mathcal{P}$ , we “travel” similarly by the inference rules, *Modus Ponens* especially [performed by a **Turing Machine TM**, the inference of which is considerable as realized by the *information transfer process* within a **Shannon Transfer Chain**  $(\mathbf{X}, \mathcal{K}, \mathbf{Y})$  described thermodynamically by a **Carnot Machine CM**].

The thermodynamic model for the consistent  $\mathcal{P}/T_{\mathcal{P}A}$  inference, from its axioms or formulas having been inferred so far, is created by the Carnot Machine’s activity, which models the inference. This whole Carnot Machine **CM** runs in the wider adiabatic system  $\mathcal{L}/\mathcal{D}_{\mathcal{L}}$  and, in fact, is, in this way, creating these states, [the **TM**’s,  $(\mathbf{X}, \mathcal{K}, \mathbf{Y})$ ’s, configurations are then modeled by the states  $\theta_i^{\mathcal{L}} \in \mathcal{D}_{\mathcal{L}}$  of the adiabatic  $\mathcal{L}/\mathcal{D}_{\mathcal{L}}$  with this modeling **CM** inside], see the Figure 2.

The  $\mathcal{L}$ ’s initial imbalance starts the  $\theta_{\square}^{\mathcal{L}}$ ’s states’ sequence on a trajectory  $l_{\mathcal{D}_{\mathcal{L}}}$  and is given by the modeled

temperature difference  $T_w - T_0 > 0$  on **CM**,  
existence of the input message on  $\mathcal{K}$ ,  
input chain’s existence on the **TM**’s input-output tape

These adiabatic trajectories  $l_{\mathcal{D}_{\mathcal{L}}}$  now represent the norm of the consistency (and resultativity) of the  $\mathcal{P}/T_{\mathcal{P}A}$ -inference/computing process expressible also in terms of the **information transfer/heat energy transformation**.

♣ The adiabatic property of the thermodynamic system  $\mathcal{L}$  is always created over the given scales of its state quantities—over their scale for a certain “creating” original (and not adiabatic) system  $\mathcal{T}$ , and by its *outerly specification or the design/construction* by means of **heat/adiabatic isolation of the space  $V_{max}$  of the original system  $\mathcal{T}$  that the system  $(\mathcal{L}/\mathcal{T})$  can occupy**, and after the system  $\mathcal{L}$  has been (as the adiabatic isolated original system  $\mathcal{T}$ ) **designed and set in the starting state  $\theta_1$** , see Figure 1. The state  $\theta_4$  is a state  $\diamond$  of the set of states  $\{\diamond\}$ . These states are those ones in the Figure 1, which, although they are in the given scale of state quantities  $U$  and  $V$  of the state space  $\mathcal{D}_{\mathcal{L}}$  of the system  $\mathcal{L}$  considered,  $U \in \langle U_{min}, U_{max} \rangle$  and  $V \in \langle V_{min}, V_{max} \rangle$ , are within it [in (the state space  $\mathcal{D}_{\mathcal{L}}$  of)  $\mathcal{L}$ ]

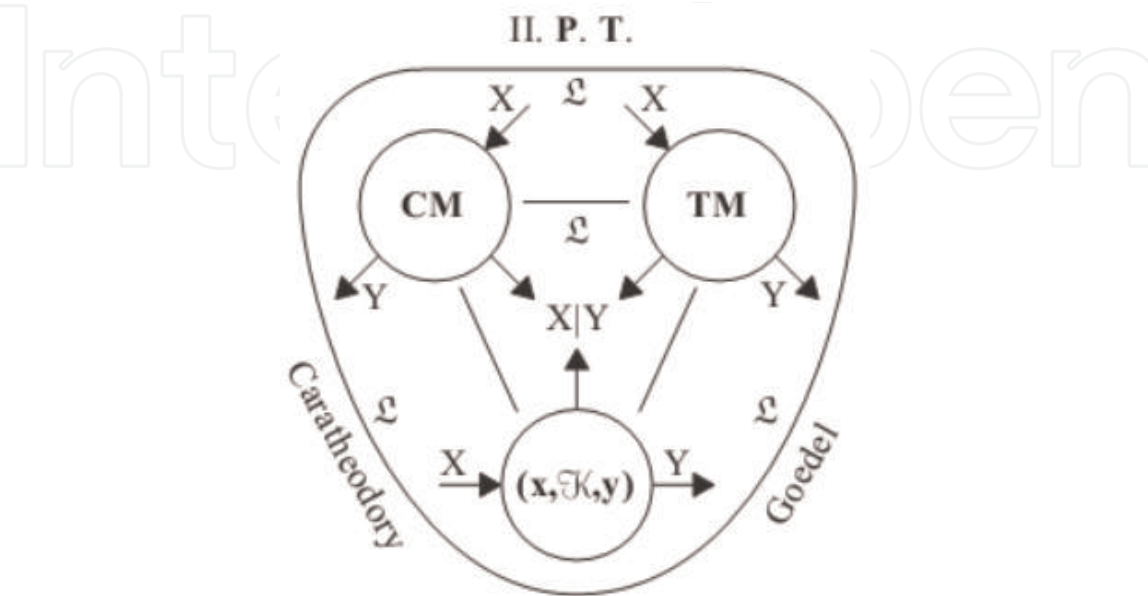


Figure 2.  
The mutual describability of the **CM**,  $(\mathbf{X}, \mathcal{K}, \mathbf{Y})$  and **TM**.

**by permitted** (adiabatic,  $dQ_{Ext} = 0$ ) **changes**  $l_{2b}, l_{2b'}, l_{2d}, l_{2e}$  and  $l_3$ , **inaccessible**. And certainly, thermodynamic states  $\square$  beyond these scales, within the hierarchically higher systems, are not accessible from the inside of the system  $\mathcal{L}/\mathcal{T}$  itself, without its (not adiabatical) widening, either, see the **Figure 1**.

. Without violation of the adiabacity of the system  $\mathcal{L}$ , it is not possible to reach the state  $\theta_4$  from the state  $\theta_1$  along any simple path  $l_{2b}, l_{2b'}, l_{2d}, l_{2e}$  in the state space  $\mathcal{D}_{\mathcal{L}}$ ,

♣ However, **outside the adiabacity of the system  $\mathcal{L}$**  expressed by the relation  $dQ_{Ext} = 0$ , which means **under the opposite requirement  $dQ_{Ext} \neq 0$ , it is possible to design or to construct a (nonadiabatic) path linking a certain point/state of the state space  $\mathcal{D}_{\mathcal{L}}$  located, e.g., on  $l_{2e}$  with the point/state  $\theta_4$ ; for example, it is the path  $l_4$  from  $\theta_1$  to  $\theta_4$ , now in a certain nonadiabatic system  $\mathcal{N}$ ,  $\mathcal{N} \subseteq \mathcal{T}$  where, from the view of possibilities of changes of the state, see **Figure 1**, is valid that**

$$\begin{aligned} \square \notin \mathcal{D}_{\mathcal{L}/\mathcal{T}}, \{\square\} \not\subseteq \mathcal{D}_{\mathcal{L}/\mathcal{T}}; \quad \diamond \notin \{l_{2b}, l_{2b'}, l_{2d}, l_{2e}, l_3\}, \quad \{\diamond\} \not\subseteq \mathcal{D}_{\mathcal{L}/\mathcal{T}} \\ \mathcal{D}_{\mathcal{L}} = \mathcal{D}_{\mathcal{N}} = \mathcal{D}_{\mathcal{T}} \triangleq \mathcal{D}_{\mathcal{L}/\mathcal{T}} \quad \mathcal{T} \supseteq \mathcal{N} \supseteq \mathcal{L} \end{aligned} \quad (9)$$

. Further, it is possible to create for this nonadiabatic system  $\mathcal{N}$  an alternative adiabatic system  $\mathcal{L}'$  ( $\mathcal{D}_{\mathcal{L}'} \supseteq \mathcal{D}_{\mathcal{L}}$ ) enabling adiabatic-isochoric changes, e.g.,  $\theta_{2e} \rightarrow \theta_4$ .

.. Both the new adiabatic system  $\mathcal{L}'$  and its nonadiabatic “model”  $\mathcal{N}$  can be a subsystem of another but also adiabatic and imminently superior system  $\mathcal{L}^+$  having another/wider range of the state quantities than it was for the original systems  $\mathcal{L}$  and  $\mathcal{N}$ , ( $\mathcal{D}_{\mathcal{L}} \subseteq \mathcal{D}_{\mathcal{L}'/\mathcal{N}} \subseteq \mathcal{D}_{\mathcal{L}^+}$ ). Then the path  $l_4$  in the state space  $\mathcal{D}_{\mathcal{L}'/\mathcal{N}}$  of the system  $\mathcal{L}'/\mathcal{N}$  will be, from the point of  $\mathcal{L}'$  of the imminently superior adiabatic system  $\mathcal{L}^+$ , the adiabatic one—the system  $\mathcal{L}'$  is already isolated in  $\mathcal{L}^+$  and the system  $\mathcal{L}^+$  itself is already created in a certain system  $\mathcal{L}^*$  imminently superior to it, as an isolated/adiabatic substitute for the system  $\mathcal{N}'$  ( $\mathcal{D}_{\mathcal{L}'/\mathcal{N}} \subsetneq \mathcal{D}_{\mathcal{L}^+}, \mathcal{D}_{\mathcal{L}^+/\mathcal{N}'} \subsetneq \mathcal{D}_{\mathcal{L}^*} \dots$ ).

.. From the view of the *possibilities to change the state*, or from the view of the *energetic relations* ( $\mathcal{E}$ ), it is possible, see the **Figure 1**, to write,

$$\mathcal{L} \subsetneq \mathcal{L}' \subsetneq \mathcal{L}^+ \subsetneq \mathcal{L}^* \subsetneq \mathcal{L}^{**} \dots, \quad \mathcal{N} \subsetneq \mathcal{N}' \subsetneq \mathcal{N}^+ \subsetneq \mathcal{N}^* \subsetneq \mathcal{N}^{**} \dots \quad (10)$$

$\{\mathcal{N}' \subsetneq \mathcal{L}^+\}_{\mathcal{E}}$ ,  $\mathcal{N}$  is implemented in  $\mathcal{L}'$ ,  $\{\mathcal{N}^+ \subsetneq \mathcal{L}^*\}_{\mathcal{E}}$ ,  $\mathcal{N}'$  is implemented in  $\mathcal{L}^+$   
 $\{\mathcal{N}^+ \subsetneq \mathcal{L}^*\}_{\mathcal{E}}$ ,  $\mathcal{N}^+$  is implemented in  $\mathcal{L}^*$ , ...

We introduce a symbol  $l_{\mathcal{D}_{\mathcal{L}}[\cdot]}$  for adiabatic paths in the state spaces  $\mathcal{D}_{\mathcal{L}[\cdot]}$ ,

$$\begin{aligned} l_{\mathcal{D}_{\mathcal{L}}} \triangleq \{l_{2b}, l_{2b'}, l_{2d}, l_{2e}, l_3\}, \quad l_{\mathcal{D}_{\mathcal{L}'}} \triangleq \{l_{2b}, l_{2b'}, l_{2d}, l_{2e}, l_3, l_{2e} - l_{\theta_{2e}, \theta_4}, l_5\}, \quad l_{\mathcal{D}_{\mathcal{L}^+}}, l_{\mathcal{D}_{\mathcal{L}^*}}, \dots \\ l_{\mathcal{D}_{\mathcal{L}}} \subsetneq l_{\mathcal{D}_{\mathcal{L}'}} \subsetneq l_{\mathcal{D}_{\mathcal{L}^+}} \subsetneq l_{\mathcal{D}_{\mathcal{L}^*}} \subsetneq \dots \end{aligned} \quad (11)$$

♣ **The states from the sets  $\{\mathcal{D}_{\mathcal{L}} - l_{\mathcal{D}_{\mathcal{L}}}\}$ ,  $\{\mathcal{D}_{\mathcal{L}'} - l_{\mathcal{D}_{\mathcal{L}'}}\}$ ,  $\{\mathcal{D}_{\mathcal{L}^+} - l_{\mathcal{D}_{\mathcal{L}^+}}\}$ ,  $\{\mathcal{D}_{\mathcal{L}^*} - l_{\mathcal{D}_{\mathcal{L}^*}}\}$ , ... in the view of adiabacity and specification of the system  $\mathcal{L}$  are forming, within the hierarchy of the systems  $\mathcal{L}$ ,  $\mathcal{L}'$ ,  $\mathcal{L}^+$ ,  $\mathcal{L}^*$ , ..., a certain set  $\mathcal{D}_{\mathcal{L}}^* = \{\{\diamond\} \cup \{\square\}\}$ , which is in the framework of the system  $\mathcal{L}$  inaccessible/unachievable as a whole and also in any of its subset and member. However, the  $\mathcal{L}$ -inaccessibility (adiabatic inaccessibility, especially of  $\{\diamond\}$  in the state space  $\mathcal{D}_{\mathcal{L}/\mathcal{T}}$ ) also means existence of the paths  $l_{\mathcal{D}_{\mathcal{L}}}$  of the adiabatic system  $\mathcal{L}$ . In the sense of the domain of solution of its (the  $\mathcal{L}$ 's) state equations, they cannot be part of the functionality of  $\mathcal{L}$  (but mark it).**

#### 4. Analogy between adiabaticity and PA-inference

♣ Now the states on the adiabatic paths  $l_{\Omega_{\mathcal{L}}}$  (of changes of the state of the adiabatic system  $\mathcal{L}$ ) are considered to be the analogues of **PA-arithmetic claims/claims of the Peano Arithmetic theory**  $\mathcal{T}_{PA}$  (formulated/inferred/proved in  $\mathcal{P}$ ),  
 - **adiabaticity of the system  $\mathcal{L}$**  is the analog of **consistency** of the system  $\mathcal{P}_{[k]}$  and  
 . **the set  $l_{\Omega_{\mathcal{L}}}$  of adiabatic paths in  $\Omega_{\mathcal{L}/\mathcal{T}}$**  is an analog of **PA-theory  $\mathcal{T}_{PA}$** ; then, adiabatic analogy of the higher consistent inferential system  $\mathcal{P}'$  is by  $\mathcal{L}', \mathcal{P}' \supsetneq \mathcal{P}, \dots$   
 .. Then the given specific **adiabatic path  $l_{2b}, l_{2b'}, l_{2d}, l_{2e}, l_3$**  is an analog of certain **deducible thread  $\vec{x} Bx_k$  of the claim  $x_k$  of the theory  $\mathcal{T}_{PA}$** , where

$$\begin{aligned} \vec{x} Bx_k &= (x_1, x_2, \dots, x_{k-1}, x_k) Bx_k = "1" \\ x_1 &\in \{AXIOMS\}^{\mathcal{P}} \quad \text{and} \quad x_1 \cong \theta_1 \\ x_1, x_2, \dots, x_{k-1}, x_k &\in \mathcal{T}_{PA} \quad \text{and} \quad x_k \cong \theta \in \{\theta_{2b}, \theta_{2b'}, \theta_{2d}, \theta_{2e}, \theta_3\} \\ x_2, \dots, x_{k-1} &\cong \theta \in \{\{l_{2b} - \theta_{2b}\}, \{l_{2b'} - \theta_{2b'}\}, \{l_{2d} - \theta_{2d}\}, \\ &\quad \{l_{2e} - \theta_{2e}\}, \{l_3 - \theta_3\}\} - \theta_1 \end{aligned} \quad (12)$$

♣ The states from the space  $\Omega_{\mathcal{L}/\mathcal{T}}$  of the system  $\mathcal{L}/\mathcal{T}$  satisfying the range of values of the state quantities  $p \in \langle p_{min}, p_{max} \rangle, V \in \langle V_{min}, V_{max} \rangle, T \in \langle T_{min}, T_{max} \rangle / U \in \langle U_{min}, U_{max} \rangle$ , which are inaccessible along any of the adiabatic paths from  $l_{\Omega_L}$ , that means they are the states  $\diamond$  from the difference  $\{\Omega_{\mathcal{L}/\mathcal{T}} - l_{\Omega_L}\}$ , shortly said from  $\{\mathcal{T} - \mathcal{L}\}$ , are considered to be analogues of not PA-claims such as, e.g., the **Fermat's Last Theorem**.<sup>12</sup> So, they are **analogues of all-the-time true ("1") arithmetic but not-PA-arithmetic claims**. From the point of adiabaticity of the system  $\mathcal{L}$ , they ( $\diamond$ ) are only some thermodynamic states of its "creating" system  $\mathcal{T}$ , and they are from the common range of values of the state quantities for  $\mathcal{T}$  and  $\mathcal{L}$ . From the point of expressing possibilities it as always true

$$\mathcal{L}^{[.]} \subsetneq \mathcal{N}^{[.]} \subseteq \mathcal{T}^{[.]} \subsetneq \mathcal{T}^* \subsetneq \left\{ \Omega_{\mathcal{L}^{[.]}/\mathcal{T}^{[.]}} \right\}^* \quad (13)$$

[Symbol  $\mathcal{T}^*$  denotes thermodynamic theory as a whole and symbol  $\left\{ \Omega_{\mathcal{L}^{[.]}/\mathcal{T}^{[.]}} \right\}^*$  is a mark for a transitive and reflexive closure of the set of (any) claims about systems  $\mathcal{L}^{[.]}/\mathcal{T}^{[.]}$ .]

♣ The whole set  $\Omega_{\mathcal{L}}^*$  of states inaccessible in a given scale of state quantities of the system  $\mathcal{L}/\mathcal{T}$  along the arbitrary adiabatic path from  $l_{\Omega_L}$  in the system  $\mathcal{L}$  (states  $\diamond$ ), as well as the set of  $\mathcal{L}$ -inaccessible states  $\square$  outside this scale, see Figure 1, are considered now to be the thermodynamic bearer of analogy of the semantics of the Gödel's UNDECIDABLE PROPOSITION 17Gen r,

$$\begin{aligned} 17Gen r &\cong \Omega_{\mathcal{L}}^* \quad [= \{\{\diamond\} \cup \{\square\}\}] \\ \Omega_{\mathcal{L}}^* &= \{\Omega_{\mathcal{L}^*} - l_{\Omega_L}\}, \quad \Omega_{\mathcal{L}}^* \subsetneq \Omega_{\mathcal{L}}^* \subsetneq \left\{ \Omega_{\mathcal{L}^*}/\mathcal{T}^* \right\}^* \end{aligned} \quad (14)$$

- The states from  $\Omega_{\mathcal{L}}^*$  (from  $\{\Omega_{\mathcal{L}^*} - l_{\Omega_L}\}, \{\Omega_{\mathcal{L}^*} - l_{\Omega_{\mathcal{L}}}\}, \{\Omega_{\mathcal{L}^*} - l_{\Omega_{\mathcal{L}'}}}\}, \dots, \{\Omega_{\mathcal{L}^{**}} - l_{\Omega_{\mathcal{L}}}\}, \dots, \{\Omega_{\mathcal{L}^{**}} - l_{\Omega_{\mathcal{L}^*}}\}, \dots$ ) inaccessible by permitted changes in currently used systems  $\mathcal{L}, \mathcal{L}', \mathcal{L}^+, \mathcal{L}^*, \dots$  (within the scale of values of their state quantities and also out of this scale) confirm both existence and properties of

<sup>12</sup> Alternatively Goldbach's conjecture.

these systems  $\mathfrak{L}, \mathfrak{L}', \mathfrak{L}^+, \mathfrak{L}^*, \dots$ ; they confirm adiabacity of changes  $l_{\mathfrak{L}}, l_{\mathfrak{L}'}, l_{\mathfrak{L}^+}, l_{\mathfrak{L}^*}, \dots$  running in them.

For (to illustrate our analogy) a supposedly countable set of states along the paths  $l_{\mathfrak{L}}$  of changes of the state of the system  $\mathfrak{L}$  (for simplicity we can consider the isentrop  $l_{2e}$  only), the PROPOSITION 17Gen  $r$  is a claim of countability set nature, the analog  $\mathfrak{Q}_{\mathfrak{L}}^*$  of which is formulated in the set  $\{\mathfrak{Q}_{\mathfrak{L}/\mathfrak{T}}\}^*$ ; it is valid that

$$\begin{aligned} \{\mathfrak{Q}_{\mathfrak{L}/\mathfrak{T}}\}^* \not\supseteq \mathfrak{Q}_{\mathfrak{L}} \quad \text{and} \quad \{\mathfrak{Q}_{\mathfrak{L}} - l_{\mathfrak{L}}\} \not\subseteq \mathfrak{Q}_{\mathfrak{L}}^* \not\subseteq \{\mathfrak{Q}_{\mathfrak{L}/\mathfrak{T}}\}^* \\ \{\mathfrak{Q}_{\mathfrak{L}} - l_{\mathfrak{L}}\} \not\supseteq \mathfrak{Q}_{\mathfrak{L}} \quad \text{and} \quad \{\mathfrak{Q}_{\mathfrak{L}} - l_{\mathfrak{L}}\} \not\supseteq l_{\mathfrak{L}} \end{aligned} \quad (15)$$

#### 4.1 Analogy between Caratheodory and Gödel theorems

We claim that, II. **Caratheodory theorem**,

◇ if an arbitrary Pfaff form  $\delta Q_{\text{Ext}} = \sum_{i=1}^n X_i dx_i$ , where  $X_i$  are functions of  $n$  variables, continuously differentiable (over a simply continuous domain) has such a quality that in the arbitrary vicinity of arbitrarily chosen fixed point  $P$  of the hyperplane  $\mathcal{R}[P \in \mathcal{R}, \mathcal{R}[(x_i)_{i=1}^n] = C = \text{const.}]$  there exists a set of points inaccessible from the point  $P$  along the path satisfying the equation  $dQ_{\text{Ext}} = 0$ , then it is possible to find an integration factor for it and then this form is holonomous. In a physical sense and, by means of the Thermodynamics language,

$$(\exists l_{\mathfrak{L}}) \Rightarrow (\exists \mathfrak{Q}_{\mathfrak{L}}) \Rightarrow (\exists \mathfrak{Q}_{\mathfrak{L}}^*) \Rightarrow (\exists \mathfrak{Q}_{\mathfrak{L}^*}) \Rightarrow \left( \exists \{\mathfrak{Q}_{\mathfrak{L}/\mathfrak{T}}\}^* \right) \quad (16)$$

it says what, in its consequence [w 17Gen  $r$ , (8)] and in a meta-arithmetic-logical way, the II. **Gödel theorem** (corrected semantically by [3, 9, 10]) claims;

♣ if  $\kappa$  is an arbitrary recursive and consistent CLASS OF FORMULAE, then any CLAIM (written as the SENTENCIAL and as such, representing a countable set of claims, which are its implementations) saying that CLASS  $\kappa$  is consistent must be constructed outside this set and for this fact it is not  $\kappa$ -PROVABLE/is  $\kappa$ -UNPROVABLE or cannot be  $\kappa$ -PROVABLE. In fact, it is a part of the inconsistent metasystem  $\mathcal{P}^*$ .

- Outside the consistent system  $\mathcal{P}_{\kappa}$ , there is a true (“1”) formula whose ARITHMETIZATION is  $\kappa$ -UNPROVABLE FORMULA/PROPOSITION/CLAIM or code 17Gen  $r$ .<sup>13</sup>

. In a physical sense and by the Thermodynamics language,

$$\{\mathfrak{Q}_{\mathfrak{L}/\mathfrak{T}}\}^* \not\subseteq \mathfrak{Q}_{\mathfrak{L}}^* \not\subseteq \mathfrak{Q}_{\mathfrak{L}}^* \not\subseteq \mathfrak{Q}_{\mathfrak{L}} \not\subseteq l_{\mathfrak{L}} \quad (17)$$

♣ It is possible to claim that, I. **Caratheodory theorem**,

◇ if an arbitrary Pfaff form  $\delta Q_{\text{Ext}} = \sum_{i=1}^n X_i dx_i$  has a integration factor, then there are in the arbitrary vicinity of an arbitrarily chosen fixed point  $P$  of the hyperplane  $\mathcal{R}$  some points inaccessible from this point  $P$   $[P \in \mathcal{R}[(x_i)_{i=1}^n] = \text{const.}]$  along the path satisfying the equation  $dQ_{\text{Ext}} = 0$ . In a physical sense and by means of the Thermodynamics language,

$$[\mathfrak{Q}_{\mathfrak{L}}^* \not\subseteq l_{\mathfrak{L}}] \wedge \left[ \left( \{\mathfrak{Q}_{\mathfrak{L}/\mathfrak{T}}\}^* - \mathfrak{Q}_{\mathfrak{L}}^* \right) \not\subseteq l_{\mathfrak{L}} \right] \quad (18)$$

<sup>13</sup> Any attempt to prove/TO PROVE it (to infer/TO INFER it) within the system  $\mathcal{P}_{\kappa}$  assumes or leads to the *Circulus Vicius*.



**it says what**, in a meta-arithmetic-logical way, the **I. Gödel theorem** (corrected semantically by [3, 9, 10]) **claims**;

♣ for every *recursive* and *consistent* CLASS OF FORMULAE  $\kappa$  and outside this set, there is such a true (“1”) CLAIM  $r$  with free VARIABLE  $v$  [ $r \triangleq r(v)$ ] that neither PROPOSITION  $vGen\ r$  nor PROPOSITION  $Neg(vGen\ r)$  belongs to the set  $Flg(\kappa)$ ,

$$[vGen\ r \notin Flg(\kappa)] \ \& \ [Neg(vGen\ r) \notin Flg(\kappa)] \quad (19)$$

FORMULA  $vGen\ r$  and  $Neg(vGen\ r)$  are not  $\kappa$ -PROVABLE—FORMULA  $vGen\ r$  is not  $\kappa$ -DECIDABLE. They are elements of inconsistent (meta)system  $\mathcal{P}^*$ .

♣ **For us, as an isolated system  $\mathcal{L}$ , to achieve such a “state,”** it is necessary to consider the states with values of state quantities which are not a part of the domain of solution of the state equation for  $\mathcal{L}$ . The system  $\mathcal{L}$  has not been designed for them (so, we are facing inconsistency). For example, **the required volume  $V$  and temperature  $T$  should be greater than their maxima  $V_{max}$  and  $T_{max}$  achievable by the system  $\mathcal{L}$ .** In order “to achieve” them, **the system  $\mathcal{L}$  itself would have to “get out of itself,” and in order to obtain values  $V$  and  $T$  greater than  $V_{max}$  and  $T_{max}$ , it would have to “redesign”/reconstruct itself.** However, it is **us, being in a position of the hierarchically higher object**, who has to do so, from the outside the state space  $\mathcal{Q}_{\mathcal{L}/\mathcal{T}}$  (from the outside the volume  $V_{max}$ ), which the system may occupy now.<sup>14</sup>

- This “procedure” corresponds to the CLAIM/PROPOSITION/FORMULA  $17Gen\ r$  construction by means of (Cantor’s) **diagonal argument** and **Caratheodory proof**.

♣ The states unachievable within the state spaces of the systems  $\mathcal{L}, \mathcal{L}', \mathcal{L}^+, \mathcal{L}^*, \dots$  or inaccessible from them are creating, as a whole, a *certain class of equivalence or macrostate*  $\mathcal{Q}_{\mathcal{L}^{[.]}}^*$  ( $\mathcal{Q}_{\mathcal{L}^{[.]}}^* \not\subseteq \mathcal{Q}_{\mathcal{L}^{[.]}} \not\subseteq \mathcal{Q}_{\mathcal{L}^{[.]}}$ ) in hierarchy of the state spaces, from the point of their possible development, of always superior systems — $\{\mathcal{L}' \cup \mathcal{L}^+ \cup \mathcal{L}^*\}$  for  $\mathcal{L}$ ,  $\{\mathcal{L}^+ \cup \mathcal{L}^*\}$  for  $\mathcal{L}'$ ,  $\mathcal{L}^*$  for  $\mathcal{L}^+$ ,  $\dots$ . The existence of the macrostate  $\mathcal{Q}_{\mathcal{L}^{[.]}}^*$ , already beginning from the original system  $\mathcal{L}$  (macrostate  $\mathcal{Q}_{\mathcal{L}}^*$ ), confirms the existence of the currently considered (adiabatic) system  $\mathcal{L}^{[.]}$  and its properties, especially its adiabaticity. And by this, in our analogy, it also confirms the consistency of its arithmetic/mathematical analog  $\mathcal{P}, \mathcal{P}', \mathcal{P}^+, \dots$  (a complement of the set cannot exist without this set) and, on the contrary,

$$\begin{aligned} (\exists |\{\mathcal{Q}_{\mathcal{L}/\mathcal{T}}\}^*) &\Rightarrow (\exists |\mathcal{Q}_{\mathcal{L}^{[.]}}^*), & (\exists |\{\mathcal{Q}_{\mathcal{L}/\mathcal{T}}\}^*) &\Leftarrow (\exists |\mathcal{Q}_{\mathcal{L}^{[.]}}^*); \\ \left[ \left[ (\exists |\mathcal{Q}_{\mathcal{L}^{[.]}}^*) &\Rightarrow (\exists |\mathcal{L}^{[.]}) \right] \right. & \& \left. \left[ (\exists |\mathcal{L}^{[.]}) &\Rightarrow (\exists |\mathcal{Q}_{\mathcal{L}^{[.]}}^*) \right] \right] & (20) \\ &\Rightarrow & & \\ & \left[ (\exists |\mathcal{Q}_{\mathcal{L}^{[.]}}^*) &\Leftrightarrow (\exists |\mathcal{L}^{[.]}) \right] \end{aligned}$$

- Based just upon this point of view, we assign the set/macrostate or equivalence class  $\mathcal{Q}_{\mathcal{L}^{[.]}}^*$  the meaning of the bearer of the sense of the **Gödel’s UNDECIDABLE PROPOSITION  $17Gen\ r$  for  $\mathcal{P}$** ,

<sup>14</sup> This also involves introduction of the representative  $\theta_0$  of Fermat’s Last Theorem provided we are speaking about  $\mathcal{L}$  with  $l_{\mathcal{Q}_{\mathcal{L}}}$  and provided we require enlargement  $\mathcal{L}'$  in order to get  $\mathcal{L}' \cong \mathcal{P}'$ .

The specific states accessible in the state space  $\mathcal{Q}_{\mathcal{L}} = \{p \in \langle p_{min}, p_{max} \rangle, V \in \langle V_{min}, V_{max} \rangle,$

$T \in \langle T_{min}, T_{max} \rangle / U \in \langle U_{min}, U_{max} \rangle, \dots$  of the isolated system  $\mathcal{L}$  through reversible or irreversible changes other than adiabatic are thermodynamic analogy (interpretation) of the enlargement of the axiomatics of the original system  $\mathcal{P}_{[k]}$  to the new system  $\mathcal{P}', \mathcal{P}^+, \dots$ , *similar/relative* to the  $\mathcal{P}_{[k]}$ . Such an enlargement of the system  $\mathcal{P}$  to a certain system  $\mathcal{P}^{[.]}$  enabled Andrew Wiles to prove the Fermat’s Last Theorem.

Through its representative  $\theta_0$  we enlarge  $\mathcal{L}$  to  $\mathcal{L}'$ ,  $\mathcal{L}' \cong \mathcal{P}'$ .



- the  $\mathcal{L}$ -unachievability of the set  $\mathfrak{Q}_{\mathcal{L}}^*$  is in the position of the analog for this, in fact, methodological axiom which has been formulated in a certain hierarchically higher inferential (meta)system  $\mathcal{P}^*$ ,  $\mathcal{P}^* \cong \{\mathfrak{Q}_{\mathcal{L}}\}^*$ . In accordance with the above and with **Figure 1**, we write for  $\mathcal{L}/\mathcal{P}$

$$\begin{aligned} \theta_4 \in \{\mathfrak{Q}_{\mathcal{L}}^* - l_{\mathfrak{Q}_{\mathcal{L}}}\} &= \{\{\Diamond\} \cup \{\Box\}\} = \mathfrak{Q}_{\mathcal{L}}^* \subsetneq \{\{\mathfrak{Q}_{\mathcal{L}}^*\}^* - l_{\mathfrak{Q}_{\mathcal{L}}}\} \subsetneq \{\mathfrak{Q}_{\mathcal{L}}\}^* \\ [l_{\mathfrak{Q}_{\mathcal{L}}} \not\models \theta_4] &\Rightarrow [l_{\mathfrak{Q}_{\mathcal{L}}} \not\models \{\mathfrak{Q}_{\mathcal{L}}^* - l_{\mathfrak{Q}_{\mathcal{L}}}\}] \quad [\in \{\{\mathfrak{Q}_{\mathcal{L}}^*\}^* - l_{\mathfrak{Q}_{\mathcal{L}}}\}] \\ [l_{\mathfrak{Q}_{\mathcal{L}}} \not\models \{\mathfrak{Q}_{\mathcal{L}}^* - l_{\mathfrak{Q}_{\mathcal{L}}}\}] &\cong [l_{\mathfrak{Q}_{\mathcal{L}}} \not\models \{\mathfrak{Q}_{\mathcal{L}}^* - l_{\mathfrak{Q}_{\mathcal{L}}}\}] \\ [l_{\mathfrak{Q}_{\mathcal{L}}} \not\models [l_{\mathfrak{Q}_{\mathcal{L}}} \not\models \{\mathfrak{Q}_{\mathcal{L}}^* - l_{\mathfrak{Q}_{\mathcal{L}}}\}]] &[\in \{\{\mathfrak{Q}_{\mathcal{L}}^*\}^* - l_{\mathfrak{Q}_{\mathcal{L}}}\}] \\ [l_{\mathfrak{Q}_{\mathcal{L}}} \not\models [l_{\mathfrak{Q}_{\mathcal{L}}} \not\models [l_{\mathfrak{Q}_{\mathcal{L}}} \not\models \{\mathfrak{Q}_{\mathcal{L}}^* - l_{\mathfrak{Q}_{\mathcal{L}}}\}]]] &[\in \{\{\mathfrak{Q}_{\mathcal{L}}^*\}^* - l_{\mathfrak{Q}_{\mathcal{L}}}\}], \dots \end{aligned} \quad (21)$$

and further, for the theory  $l_{\mathfrak{Q}_{\mathcal{L}}}/\mathcal{T}_{\mathcal{P}\mathcal{A}}$ , following (1)–(6) and [4], we write

$$\begin{aligned} l_{\mathfrak{Q}_{\mathcal{L}}} &\cong \mathcal{T}_{\mathcal{P}\mathcal{A}}, \quad \text{card } l_{\mathfrak{Q}_{\mathcal{L}}} = \text{card } \mathcal{T}_{\mathcal{P}\mathcal{A}} = \aleph_0 \\ \text{card } \{\mathfrak{Q}_{\mathcal{L}}^*\}^* &= \aleph_1, \quad \text{card } \{\{\mathfrak{Q}_{\mathcal{L}}^*\}^* - l_{\mathfrak{Q}_{\mathcal{L}}}\} = 1 \\ l_{\mathfrak{Q}_{\mathcal{L}}} &\cong 17, \quad \{\{\Diamond\} \cup \{\Box\}\} \cong 19 \quad [19 \in \{\{\mathfrak{Q}_{\mathcal{L}}^*\}^* - l_{\mathfrak{Q}_{\mathcal{L}}}\}] \\ y = q[17, 19] &\cong [\theta_{[\cdot]} \not\models \{\{\Diamond\} \cup \{\Box\}\}], \quad p = 17\text{Gen } q[17, 19] \\ y[Z(y)] = q[17, y] &\cong [\theta_{[\cdot]} \not\models [\theta_{[\cdot]} \not\models \{\{\Diamond\} \cup \{\Box\}\}]] \\ [[\theta_{[\cdot]} \not\models (\theta \cup \Box)] &\in \{\{\theta\} \cup \{\Box\}\}^*] \\ [\theta_{[\cdot]} \not\models [\theta_{[\cdot]} \not\models \{\{\Diamond\} \cup \{\Box\}\}]] &[\in \{\{\Diamond\} \cup \{\Box\}\}^*] \\ [\theta_{[\cdot]} \not\models [\theta_{[\cdot]} \not\models [\theta_{[\cdot]} \not\models \{\{\Diamond\} \cup \{\Box\}\}]]] &, \dots \end{aligned} \quad (22)$$

For  $19 := Z(p)$  is  $p[Z(p)] = r(17)$  and  $r(17) \cong \{\{\Diamond\} \cup \{\Box\}\}^*$  and so we can write neatly

$$\begin{aligned} [\forall \theta_{[\cdot]} \in l_{\mathfrak{Q}_{\mathcal{L}}} &\cong 17\text{Gen}, \quad [\forall \theta_{[\cdot]} \in l_{\mathfrak{Q}_{\mathcal{L}}} | [\theta_{[\cdot]} \not\models \{\{\Diamond\} \cup \{\Box\}\}]] \cong [17\text{Gen } q(17, 19)] \\ [17\text{Gen } r] &\cong [\forall \theta_{[\cdot]} \in l_{\mathfrak{Q}_{\mathcal{L}}} | \theta_{[\cdot]} \not\models [\forall \theta_{[\cdot]} \in l_{\mathfrak{Q}_{\mathcal{L}}} | \theta_{[\cdot]} \not\models \{\{\Diamond\} \cup \{\Box\}\}]] \\ [17\text{Gen } r] &\cong [l_{\mathfrak{Q}_{\mathcal{L}}} \not\models [l_{\mathfrak{Q}_{\mathcal{L}}} \not\models \{\mathfrak{Q}_{\mathcal{L}}^* - l_{\mathfrak{Q}_{\mathcal{L}}}\}]], \dots \end{aligned} \quad (23)$$

which is the same as (21).

- It is obvious from our thermodynamic analogy that *CLAIM/PROPOSITION*  $17\text{Gen } r$  for has to be true and **in connection with Gödel's II. theorem, and in accordance with Caratheodory we claim that**

$$\mathfrak{Q}_{\mathcal{L}}^* \cong 17\text{Gen } r \text{ for } \mathcal{L}/\mathcal{P} \quad [v\text{Gen } r \text{ for } \mathcal{L}'/\mathcal{P}', \mathcal{L}^+, \dots] \quad (24)$$

♣ **The notation**  $17\text{Gen } r$  itself expresses the property of the system  $\mathcal{P}$  and also the theory  $\mathcal{T}_{\mathcal{P}\mathcal{A}}$ , just as an *subject* which itself is not and cannot be the object of its own, and thus its notation is **not and cannot be one of the objects of the system**  $\mathcal{P}$  [similarly, as (17) is valid,  $\mathfrak{Q}_{\mathcal{L}}^* \not\subseteq \mathfrak{Q}_{\mathcal{L}} \not\subseteq l_{\mathfrak{Q}_{\mathcal{L}}}$ ].

**Demonstration:** Following (8)  $[Wid(\mathcal{P}) \Rightarrow 17Gen\ r]$ , we claim for the systems  $\mathcal{L}/\mathcal{P}$  that

$$\begin{aligned}
 & [dQ_{Ext}^{\mathcal{L}} = 0] \cong w, \quad [dQ_{Ext}^{\mathcal{L}} = 0] \equiv \mathcal{L}; \quad w \cong [dQ_{Ext}^{\mathcal{L}} = 0], \quad \mathcal{L} \equiv [dQ_{Ext}^{\mathcal{L}} = 0] \\
 & (\exists|\mathcal{L}) \Rightarrow (\exists|\mathcal{Q}_{\mathcal{L}}^*), (\exists|\mathcal{Q}_{\mathcal{L}}^*) \cong 17Gen\ r; \quad (\exists|\mathcal{Q}_{\mathcal{L}}^*) \Rightarrow (\exists|\mathcal{L}), (\exists|\mathcal{L}) \cong 17Gen\ r \\
 & [(\exists|\mathcal{L}) \Rightarrow (\exists|\mathcal{Q}_{\mathcal{L}}^*)] \cong [w \Rightarrow (17Gen\ r)]; \quad [(\exists|\mathcal{Q}_{\mathcal{L}}^*) \Rightarrow (\exists|\mathcal{L})] \cong [(17Gen\ r) \Rightarrow w] \\
 & \text{so, that} \quad [(\exists|\mathcal{Q}_{\mathcal{L}}^*) \Rightarrow (\exists|\mathcal{L})] \cong [(17Gen\ r) \Rightarrow w] \\
 & \& \\
 & [(\exists|\mathcal{L}) \Rightarrow (\exists|\mathcal{Q}_{\mathcal{L}}^*)] \cong [w \Rightarrow (17Gen\ r)] \text{ and then} \\
 & (\exists|\mathcal{Q}_{\mathcal{L}}^*) \equiv 17Gen\ r
 \end{aligned} \tag{25}$$

I. Gödel theorem (corrected semantically by [3, 9, 10]):

**For every recursive and consistent CLASS OF FORMULAE  $\kappa$ , and outside this set, there exists the true ("1") CLAIM  $r$  with a free VARIABLE  $v$  that neither the CLAIM  $vGen\ r$  nor the CLAIM  $Neg(vGen\ r)$  belongs to the set  $Flg(\kappa)$**

$$[vGen\ r \notin Flg(\kappa)] \& [Neg(vGen\ r) \notin Flg(\kappa)],$$

**CLAIMS  $vGen\ r$  and  $Neg(vGen\ r)$  are not  $\kappa$ -PROVABLE, the CLAIM  $vGen\ r$  is not  $\kappa$ -DECIDABLE.**

[They are elements of the formulating/syntactic metasystem  $\kappa^*$ , inconsistent against  $\kappa$ ].

II. Gödel theorem (corrected semantically by [3, 9, 10]):

**If  $\kappa$  is an arbitrary recursive and consistent CLASS OF FORMULAE, then any CLAIM saying that CLASS  $\kappa$  is consistent must be constructed outside this set and for this fact, it is not  $\kappa$ -PROVABLE.**

The consistency of the CLASS OF FORMULAE  $\kappa$  is tested by the relation  $Wid(\kappa)$ .

$$Wid(\kappa) \sim (Ex)[CLAIM(x) \& \overline{Proof_{\kappa}(x)}]$$

**The FORMULAE class  $\kappa$  is consistent.**

$\Leftrightarrow$

**at least one  $\kappa$ -UNPROVABLE CLAIM  $x$  exists.**

Now  $x = 17Gen\ r \notin \mathcal{P}/\mathcal{T}_{\mathcal{P},A}, \kappa = \mathcal{T}_{\mathcal{P},A}, \mathcal{T}_{\mathcal{P},A} \subset \mathcal{P} \subset \mathcal{P}^*$

Then, **semantically understood** and with the language of logic and meta-arithmetics, the full meaning of the **Gödel proof expresses the universal validity of the II. Law of Thermodynamics.**<sup>15</sup>

$$[II.P.T.[Proof_{\mathcal{P}}(17Gen\ r) = "0"] = "1"] = Wid(\mathcal{T}_{\mathcal{P},A})$$

<sup>15</sup> Our consideration is based on the similarity between the Cantor diagonal argument used in construction of the Gödel Undecidable Formula and the proof way of the Caratheodory theorems; adiabacity/consistency is proved by leaving them and sustaining their validity - paradox.

## 5. Conclusion

Peano Arithmetic theory is generated by its inferential rules (rules of the inferential system in which it is formulated). It consists of parts bound mutually just by these rules, but none of them is not identical with it nor with the system in their totality.

By information-thermodynamic and computing analysis of Peano arithmetic proving, we have showed why the Gödel formula and its negation are not provable and decidable within it. They are constructed, not inferred, by the diagonal argument, which is not from the set of the inferential rules of the system. The attempt to prove them leads to awaiting of the end of the infinite cycle being generated by the application of the substitution function just by the diagonal argument. For this case, the substitution function is not countable and for this it is not recursive (although in the Gödel original definition is claimed that it is). We redefine it to be total by the zero value for this case. This new substitution function generates the Gödel numbers of chains, which are not only satisfying the recursive grammar of formulae but it itself is recursive. The option of the zero value follows also from the vision of the inferential process as it would be the information transfer. The attempt to prove the Gödel Undecidable Formula is the attempt of the transfer of that information, which is equal to the information expressing the inner structure of the information transfer channel. In the thermodynamic point of view, we achieve the equilibrium status, which is an equivalent to the inconsistent theory. So, we can see that the Gödel Undecidable Formula is not a formula of the Peano Arithmetics and, also, that it is not an arithmetical claim at all. From the thermodynamic consideration follows that even we need a certain effort or energy to construct it, within the frame of the theory this is irrelevant. It is the error in the inference and cannot be part of the theory and also it is not the system. Its information value in it (as in the system of the information transfer) is zero. But it is the true claim about **inferential properties of the theory** (in fact, of the **properties of the information transfer**).

Any description of real objects, no matter how precise, is only a model of them, of their properties and relations, making them available in a specified and somewhat limited (compared with the reality) point of view determined by the description/model designer. This determination is expressed in definitions and axiomatics of this description/model/theory—both with definitions and by axioms and their number. Hence, realistically/empirically or rationally, it will also be true about (objects of) reality what such a model, called *recursive and able-of-axiomatization*, does not include. With regard of reality any such a model is *axiomatically incomplete*, even if the system of axioms *is complete*. **In addition, and more importantly, this description/model of objects, of their properties and possible relations** (the theory about reality) **cannot include a description of itself** just as the object of reality defined by itself (any such theory/object is not a subject of a direct description of itself). The description/model or the theory about reality is a grammar construction with substitutes and axiomatization and, as such, it is *incomplete in the Gödelian way*—**the grammar itself does not prevent a semantical mixing; but any observed real object cannot be the subject of observation of itself and this is valid for the considered theory, just as for the object of reality, too**. No description of reality arranged from its inside or created within the theory of this reality can capture the reality completely in wholeness of its all own properties. It is impossible for the models/theories considered, independently on their axiomatization. They are limited in principle [in the *real sense* of the Gödel theorems (in the Gödelian way)].

Now, **with our better comprehension**, we can claim that the **consistency of the recursive and axiomatizable system can never be proved in it itself**,

even if the system is consistent really. The reason is that a **claim of the consistency of such a system is designable only if the system is the object of outer observation/measuring/studies, which is not possible within the system itself.** Ignoring this approach is also the reason for the formulation of the **Gibbs paradox** and **Halting Problem**. Also, our awareness of this fact results in our **full understanding** of the **meaning and proof of the Gödel theorems**, very often explained and described incomprehensibly, even inconsistently or paradoxically, **and which is parallel with the way of the Caratheodory proof of the II. Thermodynamic Principle.**

Vienna Circle, 1931-1935, logical positivists Rudolph Carnap  
and Otto von Neurath:

*"Any scientifically meaningful statement is expressible in physical terms, about a movement in the observable space and time or, if the statement is not expressible this way it is meaningful scientifically when it is convertible to a statement about a language, otherwise it is of no scientific meaning."*



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## A. Appendix

### A.1 Summarizing comparison

♣ Under the adiabacity,  $[d]Q_{\text{Ext}} = 0$ , of the system  $\mathcal{L}$ , *it is not possible to derive such a CLAIM that is stating this adiabatic supposition.* This CLAIM is constructible not adiabatically, *outside* the adiabatic  $\mathcal{L}$  only.

♣ Under the consistency of the system  $\mathcal{P}$ , *it is not possible to derive such a CLAIM that is stating this consistency supposition.* This CLAIM is constructible *purely syntactically, outside* the consistent  $\mathcal{P}$  only (in  $\mathcal{P}^* - \mathcal{P}$ ) (Figure A1).

♣ Without  $\mathcal{P}^*$  we could not know that  $\mathcal{P}$  is not self-referencing and is consistent.

Autoreference/*HALTING PROBLEM*/Self-Observation

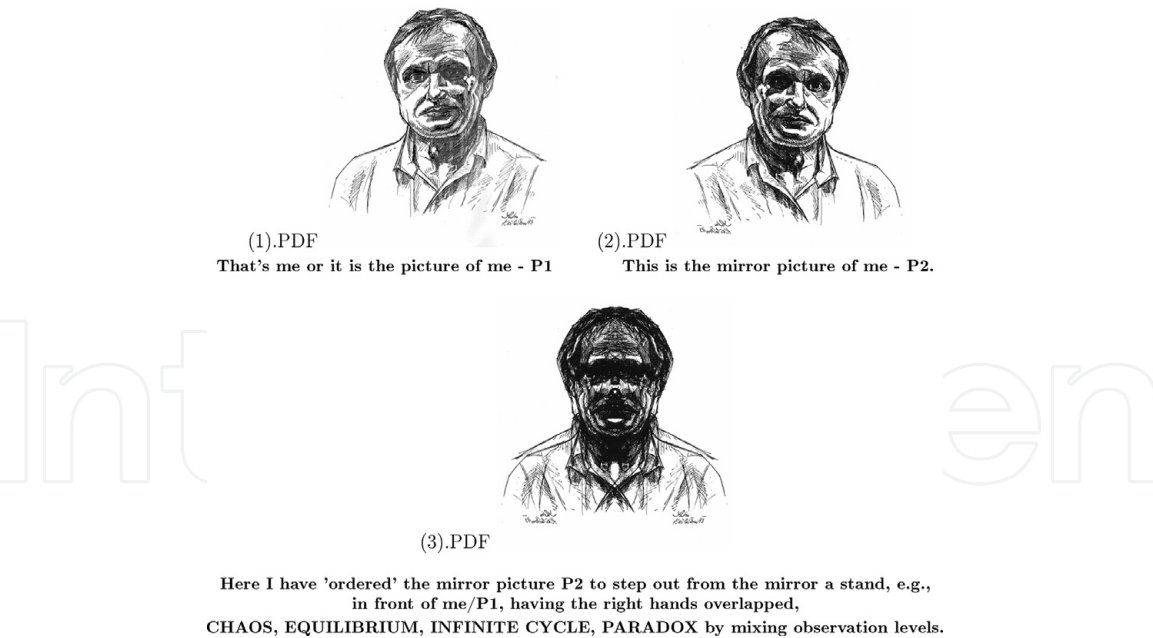
- the CLAIM about adiabacity of  $\mathcal{L}$  within  $\mathcal{L}$  -
  - the CLAIM about consistency of  $\mathcal{T}_{\mathcal{P},\mathcal{A}}$  within  $\mathcal{P}$  -
- is excluded.

This is the nature law expressed by the Caratheodory form of the II. P.T.  
and by the Gödel theorems' sense.

The eye can not look at and into itself.

Any mixing of the various observation/expressing/approach levels  
leads to the paradoxes and is to be excluded from the cognitive thinking.



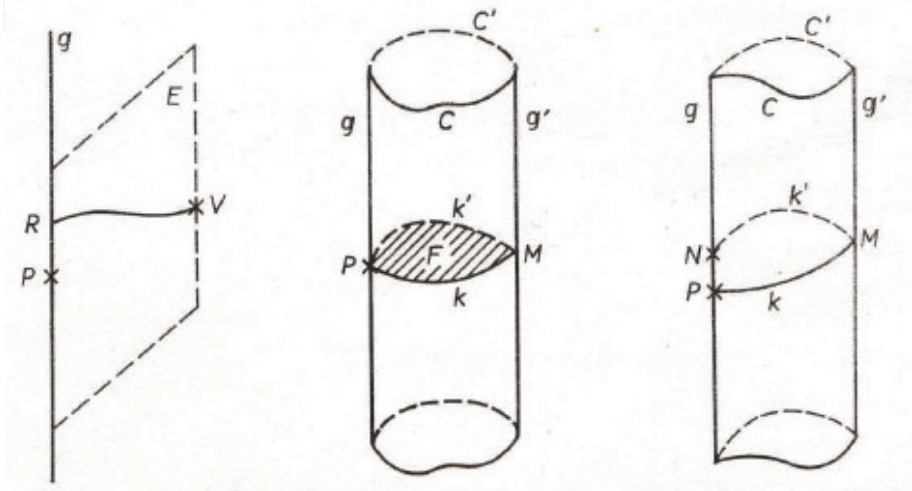


**Figure A1.**  
*Example of not distinguishing the reality and its image.*

**A.2 The proof way of Caratheodory theorems**

I. Let the form  $\delta Q = \sum_{i=1}^n X_i dx_i$  has the integration factor  $v$  and let  $d\mathcal{R} = \sum_{i=1}^n \frac{1}{v} X_i dx_i$ . Then the Pfaff equation  $\delta Q = \sum_{i=1}^n X_i dx_i = 0$  has the solution in the form  $\mathcal{R}(x_1, \dots, x_k) = \text{const.}$  and this solution represents a family of hyperplanes in  $n$ -dimensional space, not intersecting each other. **Let us pick now the point  $P(x_1^0, \dots, x_n^0)$  determined by our choice of const. = C. Only the points lying in the hyperplane  $\mathcal{R}(x_1^0, \dots, x_n^0)$  are accessible from the point P along the path satisfying the condition  $dQ = 0$ . All the points not lying in this hyperplane are inaccessible from the point P along the path satisfying the condition  $dQ = 0$  (Figure A2).**

II. Let us pick the point V, e.g., from  $\mathbb{R}^3$ , lying in a vicinity of the point P, which is not accessible from P following the path  $dQ = 0$ . Let g be a line going through the point P and let g be oriented ( $\vec{g}$ ) in such way that it does not satisfy the condition  $dQ = 0$ . The point V and the line g determine a plane  $X_i = X_i(u, v), i = 1, 2, 3$ . Let us



**Figure A2.**  
*The proof way of the Caratheodory theorems.*



consider a curve  $k$  in this plane, going through the point  $V(u_0, v_0)$  in that way ( $\vec{g}$ ) that  $dQ = 0$  is supposedly valid along this curve. **There is only one curve  $k$  for the point  $V(u_0, v_0)$ .** It lies in our plane, the plane  $X_i = X_i(u, v)$ , and then it is valid for it  $dX_i = \frac{\partial X_i}{\partial u} du + \frac{\partial X_i}{\partial v} dv$  and, considering  $dQ = 0$  along  $k$ , we get  $\sum_{i=1}^3 X_i \frac{\partial X_i}{\partial u} du + \sum_{i=1}^3 X_i \frac{\partial X_i}{\partial v} dv = 0$ .

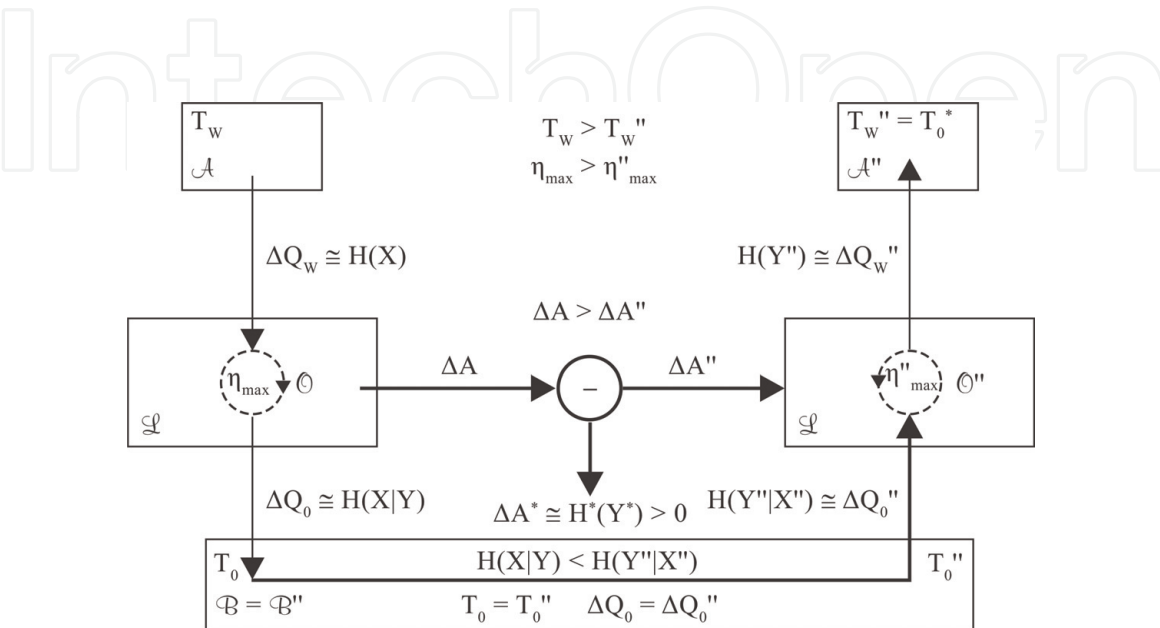
The curve  $k$ , however, intersects the line  $g$  in the point  $R$ , which is inaccessible from the point  $P$  along the path with  $dQ = 0$  (for  $dQ_{R \rightarrow g} \neq 0$ ). Otherwise, the point  $V$  would also be accessible from the point  $P$  through  $R$  and  $k$  ( $dQ_{Rk} = 0$ ), which is a conflict with the original assumption. By a suitable selection of  $V$ , it is possible to have the point  $R$  arbitrarily close to the point  $P$ ; in the arbitrary vicinity of the point  $P$ , there are points inaccessible from the point  $P$  along the path with  $dQ = 0$ . Now, let us pick a line  $g'$  parallel to the line  $g$ , and a cylinder  $C$  going through these two lines. We consider that the curve  $k$  satisfying the relation  $dQ = 0$  is on this cylinder  $C'$  goes through the point  $P$  and intersects the line  $g'$  in the point  $M$ .

Now, let us consider another cylinder  $C'$  as the continuation of  $C$  with  $g'$  and  $g$ . Let us use the symbol  $k'$  for the continuation of the curve  $k$  in  $C'$ . Then the curve  $k'$  must intersect the line  $g$  in the point  $P$ . Otherwise, it would be possible to deform the plane  $C'$  as much as to get  $C$ , thus continually merging the intersecting point  $N$  into the point  $P$  and at the moments of discrepancy of the points  $P$  and  $N$ , it would be possible to reach the point  $P$  from the point  $N$  along the line  $g$  (supposedly with  $dQ = 0$ ). However, the condition  $dQ = 0$  is not valid there ( $dQ_{R \rightarrow g} \neq 0$ ). By deforming  $C'$  into  $C$ , the  $k$  and  $k'$  would close a plane  $F$  where  $dQ = 0$ . If the equation of this plane has the form  $\mathcal{R}(x_i)_{i=1}^3 = \text{const.}$ , then the equation  $dQ = 0$  has a solution—an integration factor for the **Pfaff form**  $\delta Q = \sum_{i=1}^3 X_i dx_i$  exists [11].

### A.3 Information thermodynamic concept removing autoreference

The concept for ceasing the autoreference, based on the two Carnot Cycles disconnected as for their heaters and described informationally, shows the following **Figure A3**. (also see [1, 2, 4]):

For  $\Delta A$ , it is valid in the cycle  $\mathcal{O}''$  that



**Figure A3.**  
The concept for ceasing the autoreference.

$$\begin{aligned}\Delta A'' &= \Delta Q_W'' \cdot \left(1 - \frac{T_0}{T_W''}\right) = \Delta Q_W \cdot \frac{T_W''}{T_W} \cdot \left(1 - \frac{T_0}{T_W''}\right) = \\ &= \Delta Q_W \cdot \left(\frac{T_W''}{T_W} - \frac{T_0}{T_W}\right) = k \cdot H(X) \cdot (T_W'' - T_0) \\ &= k \cdot H(X) \cdot T_W'' \left(1 - \frac{T_0}{T_W''}\right) = k \cdot H(X) \cdot T_W'' (1 - \beta'') = k \cdot T_W'' \cdot H(Y'')\end{aligned}\quad (26)$$

and, further, for  $\Delta A$  in the cycle  $\mathcal{O}$ , we have

$$\Delta A = k \cdot H(X) \cdot T_W (1 - \beta) = k \cdot H(X) \cdot T_W \left(1 - \frac{T_0}{T_W}\right) \quad (27)$$

and thus, for the cycles  $\mathcal{O}''$  and  $\mathcal{O}$ , it is valid that

$$\begin{aligned}\frac{\Delta A''}{kT_W''} &= H(X) \cdot \left(1 - \frac{T_0}{T_W''}\right) = H(X) \cdot (1 - \beta'') = H(X) \cdot \eta_{max}'' \\ \frac{\Delta A}{kT_W} &= H(X) \cdot \left(1 - \frac{T_0}{T_W}\right) = H(X) \cdot (1 - \beta) = H(X) \cdot \eta_{max}\end{aligned}\quad (28)$$

For the whole work  $\Delta A^*$  of the combined cycle  $\mathcal{OO}''$ , we have

$$\Delta A^* = \Delta A - \Delta A'' = [kT_W \cdot H(X) \cdot (1 - \beta) - kT_W'' \cdot H(X) \cdot (1 - \beta'')] > 0 \quad (29)$$

Then, for the whole change of the thermodynamic entropy within the combined cycle  $\mathcal{OO}''$  (measured in information units *Hartley, nat, bit*) and thus for the change of the whole information entropy  $H^*(Y^*)$ , it is valid that

$$\begin{aligned}H^*(Y^*) &= \frac{\Delta A^*}{kT_W} = H(X) \cdot \left[(1 - \beta) - \frac{T_W''}{T_W} \cdot (1 - \beta'')\right] \\ &= H(X) \cdot \left(1 - \frac{T_0}{T_W} - \frac{T_W''}{T_W} + \frac{T_0}{T_W}\right) = H(X) \cdot \left(1 - \frac{T_W''}{T_W}\right)\end{aligned}\quad (30)$$

It is valid, for  $\Delta A^*$  is a *residuum work* after the work  $\Delta A$  has been performed at the temperature  $T_W$ . Evidently, the sense of the symbol  $T_W''$  (within the double cycle  $\mathcal{OO}''$  and when  $\Delta Q_0 = \Delta Q_0$ ) is expressible by the symbol  $T_0^*$ , which is possible, for the working temperatures of the whole cycle  $\mathcal{OO}''$  are  $T_W$  and  $T_W = T_0^*$ . The relation (30) expresses that fact that the double cycle  $\mathcal{OO}''$  is the direct Carnot Cycle just with its working temperatures  $T_W > T_W = T_0^*$ . In the double cycle  $\mathcal{OO}''$ , it is valid that

$$\begin{aligned}\beta'' &= \frac{\Delta Q_0''}{\Delta Q_W''} = \frac{\frac{\Delta Q_0''}{T_W''}}{\frac{\Delta Q_W''}{T_W''}} = \frac{H(Y''|X'')}{H(Y'')} = \frac{T_0}{T_W''}, T_W'' = T_0^*, \text{ cyklus } \mathcal{O}'' \\ \beta &= \frac{\Delta Q_0}{\Delta Q_W} = \frac{\frac{\Delta Q_0}{T_W}}{\frac{\Delta Q_W}{T_W}} = \frac{H(X|Y)}{H(X)} = \frac{T_0}{T_W}, \text{ cyklus } \mathcal{O} \\ \frac{\beta}{\beta''} &= \frac{T_W''}{T_W} = \frac{T_0^*}{T_W} \triangleq \beta^*\end{aligned}\quad (31)$$

and then, by (30) and (31) is writable that

$$\frac{\Delta A^*}{kT_W} = H(X) \cdot (1 - \beta^*) = H(X) \cdot \left[ 1 - \frac{H(X|Y) \cdot H(Y'')}{H(Y''|X'') \cdot H(X)} \right] > 0 \quad (32)$$

It is ensured by the propositions  $T_W > T_0$ ,  $T_0 = T_0$  and also by that fact that the loss entropy  $H(X|Y)$  is described and given by the heat  $\Delta Q_0 = \Delta Q_0''$ . But in our combined cycle  $\mathcal{OO}''$ , it is valid too that

$$H(X) = \frac{\Delta Q_W}{kT_W} = \frac{\Delta Q_W''}{kT_W''} = H(Y'') \left[ = \frac{\Delta Q_W''}{kT_0^*} \right] \quad (33)$$

and we have

$$\frac{H(X|Y)}{H(Y''|X'')} = \beta^* < 1 \quad (34)$$

For the whole information entropy  $\frac{\Delta A^*}{kT_W}$  (the whole thermodynamic entropy  $\mathcal{S}_C$  in information units) and by following the previous relations also it is valid that

$$\begin{aligned} \frac{\Delta A^*}{kT_W} &= H(Y'') - H(Y'') \cdot \beta^* = H(Y'') \cdot \left( 1 - \frac{T_0^*}{T_W} \right) \\ &= H(Y'') \cdot \left[ 1 - \frac{H(X|Y)}{H(Y''|X'')} \right] \end{aligned} \quad (35)$$

And thus, the structure of the information transfer channel  $\mathcal{K}$  [expressed by the quantity  $H(X|Y)$ ] is measurable by the value  $H^*(Y^*)$  from (32) and (35). Symbolically, we can write, using a certain growing function  $f$ ,

$$H^*(Y^*) = \frac{\Delta A^*}{kT_W} \cong f[H(X|Y)] > 0 \quad (36)$$

The cycles  $\mathcal{O}$ ,  $\mathcal{O}''$ , and  $\mathcal{OO}''$  are the Carnot Cycles, and thus from their definition and construction, they are imaginatively<sup>16</sup> in principle, the infinite cycles; in each of them the following *criterion of an infinite cycle* (see [12]) it is valid inevitably,

$$T(X^{[.]}, Y^{[.]}) = H(X^{[.]}) - H(X^{[.]}|Y^{[.]}) = H(Y^{[.]}) > 0 \text{ and } \Delta \mathcal{S}_{\mathcal{L}}^{[.]} = 0 \quad (37)$$

The construction of the cycle  $\mathcal{OO}''$  enables us to recognize that the infinite cycle  $\mathcal{O}$  is running. In our case, it is the infinite cycle from (5), (6) and also from [4, 8, 10],

$$\begin{aligned} &Q(\mathbb{X}, Y), \quad Q[\mathbb{X}, \Phi[Q(\mathbb{X}, Y)]], \quad Q[\mathbb{X}, \Phi[Q(\mathbb{X}, \Phi[Q(\mathbb{X}, Y)])]], \quad \dots \\ &Q(\mathbb{N}_0, Y), \quad Q[\mathbb{N}_0, \Phi[Q(\mathbb{N}_0, Y)]], \quad Q[\mathbb{N}_0, \Phi[Q(\mathbb{N}_0, \Phi[Q(\mathbb{N}_0, Y)])]], \quad \dots \end{aligned} \quad (38)$$

<sup>16</sup> When an infinite reserve of energy would exist.

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